THE PROBLEM OF MONITORING THE LEAKAGE OF A HEAT CARRIER

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In the approximation of slight inhomogeneity of a dielectric, the capacitance of a capacitor is determined. For a gas model, when the inhomogeneity of the dielectric results from the diffusion of an impurity into the interelectrode gap, the dynamics of the change in the capacitance and the current in a chain is studied as a function of the geometric parameters of the capacitor and the electrophysical properties of the impurity.

At the present time, in the countries of Europe, the U.S., and Russia much attention is given to the development of investment energy programs that include the creation of high-power thermoemission nuclear power plants (NPP) for work in outer space and under water [1] and superimposed low-temperature thermoemission topping plants on existing thermoelectric power stations. The basic energy-generating element in these systems is a thermoemission cesium-plasma converter, with liquid alkali metals (lithium, sodium, potassium) or their alloys being used as heat carriers. The most promising is considered to be a high-temperature outer-space NPP with a liquid-lithium heat carrier (^{7}Li is virtually not activated in fast-neutron reactors) and a starting system on the basis of a three-component sodium-potassium-cesium eutectic alloy [2].

At all stages of creation, transportation, storage, and use of a plant that contains liquid alkali metals as a working body or a heat carrier it is necessary to solve problems of safety [3]. This is due to the fact that on loss of pressure in a liquid-metal loop an uncontrolled change in the parameters of the plant and substantial local contamination of the environment by atoms of alkali metals and their aggressive poisonous compounds with components of the atmosphere occur. Timely detection of leakage makes it possible to prevent an accident at the plant, and utilization of escaping heat carrier minimizes contamination of the environment [4].

Therefore, the development of rapid methods and means for monitoring the airtightness of different-purpose liquid-metal loops is an urgent problem. Solid-state sensors of alkali-metal leakage whose operation is based on the change in the electrophysical properties of the sensing element have been investigated most extensively. However, the main disadvantage of these sensors is that they cannot monitor components that do not enter into chemical reaction with the working surface of the sensor [4].

The problem of recording the leakage of any heat carrier in any state can be solved if one continuously keeps track of the state of the environment above the monitored surface (foremost, this is weld seams and adjacent regions of vacuum fittings of liquid-metal loops), rather than the sensor surface itself, as done in well-known works. In order to record the state of the environment above the monitored surface, a single- or multilayer metal electrode that repeats the spatial configuration of the monitored surface is electrically insulated from the surface. A pseudocapacitor is employed in which the monitored surface (the wall of a heat-carrier loop) and the outer electrode are the plates of the capacitor. In case of leakage, components of the heat carrier appear in the gap, thus causing a change in the dielectric constant of the interelectrode medium and, consequently, in the capacitance of the capacitor.

To substantiate the sensitivity of the proposed method of monitoring, it is necessary to carry out a corresponding analytical investigation and a comparison of it with experimental results.

We shall restrict our considerations to a cylindrical capacitor with a small gap $(R_2 - R_1)/R_1 << 1$, which allows us to reduce the problem to a plane one:

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$$0 \le x \le d\,,\quad d = R_2 - R_1\,;$$

$$-\,l/2 \le z \le l/2\,,\quad l = 2\pi R_1\,;$$

$$-\,L/2 \le y \le L/2$$

with periodic boundary conditions with respect to the coordinate z.

As is known, the capacitance of a capacitor is equal to

$$C = \frac{q}{U}\,. \tag{1}$$

To determine the charge of the capacitor

$$q = \int_{-L/2}^{L/2} \int_{-l/2}^{l/2} \sigma\,(y,\,z)\;dydz\,, \tag{2}$$

where $\sigma(y, z)$ is the density of the surface charge of a plate [5],

$$\sigma\,(y,\,z) = -\,\frac{\varepsilon\,(0,\,y,\,z)}{4\pi}\,\frac{\partial\varphi}{\partial x}\bigg|_{x=0}\,, \tag{3}$$

it is necessary to solve the corresponding problem of electrostatics on the distribution of the potential in a capacitor with an inhomogeneous dielectric for prescribed potentials on the plates:

$$\begin{gathered} \operatorname{div}\,[\varepsilon\,(x,\,y,\,z)\ \operatorname{grad}\varphi\,] = 0\,,\quad \varphi\,(0,\,y,\,z) = \varphi_0\,, \\ \varphi\,(d,\,y,\,z) = \varphi_1\,,\quad \varphi\,(x,\,y,\,-\,l/2) = \varphi\,(x,\,y,\,l/2)\,. \end{gathered} \tag{4}$$

Along the coordinate y we will neglect end effects, and when solving system (4) we will assume that $L \to \infty$.

The dielectric permittivity will be represented in the form

$$\varepsilon\,(x,\,y,\,z) = \varepsilon_0 + \varepsilon_1\,(x,\,y,\,z)\,, \tag{5}$$

where $\varepsilon_0 = \text{const}$ is the dielectric permittivity of the basic filler, while ε_1 accounts for the presence of an impurity. We will assume that $\max \varepsilon_1 << \varepsilon_0$, which makes it possible to solve system (4) using perturbation theory:

$$\varphi\,(x,\,y,\,z) = \varphi^{(0)}\,(x,\,y,\,z) + \varphi^{(1)}\,(x,\,y,\,z)\,,\quad \varphi^{(1)} << \varphi^{(0)}\,, \tag{6}$$

and then $\varphi^{(0)}$ and $\varphi^{(1)}$ are determined by the solutions of the following boundary-value problems:

$$\begin{gathered} \Delta\varphi^{(0)} = 0\,,\quad \varphi^{(0)}\,(0,\,y,\,z) = \varphi_0\,,\quad \varphi^{(0)}\,(d,\,y,\,z) = \varphi_1\,, \\ \varphi^{(0)}\,(x,\,y,\,-\,l/2) = \varphi^{(0)}\,(x,\,y,\,l/2) \end{gathered} \tag{7}$$

and

$$\begin{gathered} \Delta\varphi^{(1)} = -\,\frac{1}{\varepsilon_0}\operatorname{div}\,[\varepsilon_1\ \operatorname{grad}\varphi^{(0)}\,]\,,\quad \varphi^{(1)}\,(0,\,y,\,z) = \varphi^{(1)}\,(d,\,y,\,z) = 0\,, \\ \varphi^{(1)}\,(x,\,y,\,-\,l/2) = \varphi^{(1)}\,(x,\,y,\,l/2)\,. \end{gathered} \tag{8}$$

The solution of (7) has the form

$$\varphi^{(0)}(x, y, z) = \varphi_0 + \frac{\varphi_1 - \varphi_0}{d} x, \tag{9}$$

and with allowance for the boundary conditions, $\varphi^{(1)}$ will be represented as

$$\varphi^{(1)}(x, y, z) = \sum_{m=-\infty}^{\infty} \int_{-\infty}^{\infty} \bar{\varphi}_m(x, k) \exp\left(i \frac{2\pi}{l} mz + iky\right) dk, \tag{10}$$

and then $\bar{\varphi}_m$ satisfies the system

$$\frac{d^2\bar{\varphi}_m}{dx^2} - \left(k^2 + \frac{4\pi^2 m^2}{l^2}\right) \bar{\varphi}_m = \frac{1}{\varepsilon_0} \frac{d\varphi^{(0)}}{dx} \frac{d\varepsilon_{1m}}{dx},$$

$$\bar{\varphi}_m(0, k) = \bar{\varphi}_m(d, k) = 0,$$

where

$$\varepsilon_{1m}(x, k) = \frac{1}{2\pi l} \int_{-\infty}^{\infty} \exp(-iky)\, dy \int_{-l/2}^{l/2} dz \exp\left(-i \frac{2\pi}{l} mz\right) \varepsilon_1(x, y, z), \tag{11}$$

and for $\bar{\varphi}_m$ we will have

$$\bar{\varphi}_m(x, k) = -\frac{1}{\varepsilon_0} \frac{d\varphi^{(0)}}{dx} \left[\int_0^x \varepsilon_{1m}(x, k) \operatorname{ch} \bar{k}(x - x')\, dx' - \right.$$

$$\left. - \frac{\operatorname{sh} \bar{k}x}{\operatorname{sh} \bar{k}d} \int_0^d \varepsilon_{1m}(x, k) \operatorname{ch} \bar{k}(d - x)\, dx \right], \tag{12}$$

$$\overline{k^2} = k^2 + \frac{4\pi^2 m^2}{l^2}.$$

Correspondingly, the density of the surface charge is equal to

$$\sigma(y, z) = -\frac{\varepsilon_0}{4\pi} \left[\frac{\partial\varphi^{(0}}{\partial x} + \frac{\partial\varphi^{(1}}{\partial x}\right]_{x=0} - \frac{\varepsilon_1(0, y, z)}{4\pi} \frac{\partial\varphi^{(0}}{\partial x}\bigg|_{x=0} =$$

$$= -\frac{\varepsilon_0}{4\pi} \frac{U}{d} \left[1 + \sum_{m=-\infty}^{\infty} \exp\left(i \frac{2\pi}{l} mz\right) \int_{-\infty}^{\infty} k \exp(iky)\, dk \times \right.$$

$$\left. \times \int_0^d \frac{\varepsilon_{1m}(x, k)}{\varepsilon_0} \frac{\operatorname{ch} \bar{k}(d - x)}{\operatorname{sh} \bar{k}d}\, dx \right], \tag{13}$$

and, with allowance for Eqs. (1)-(3), (9), (13), for the capacitance of the capacitor we obtain

$$\frac{C - C_0}{C_0} = \frac{\Delta C}{C_0} = \frac{1}{\varepsilon_0 S} \sum_{m=-\infty}^{\infty} \int_{-L/2}^{L/2} dy \int_{-l/2}^{l/2} dz \exp\left(i \frac{2\pi}{l} mz\right) \int_{-\infty}^{\infty} \times$$

$$\times k \exp(iky)\, dk \int_0^d \frac{\varepsilon_{1m}(x, k)}{\varepsilon_0} \frac{\operatorname{ch} \bar{k}(d - x)}{\operatorname{sh} \bar{k}d}\, dx, \tag{14}$$

$$S = Ll\,, \quad C_0 = \frac{\varepsilon_0 S}{4\pi d}\,.$$

Hereafter, we will confine ourselves to a gas model. The specific polarization of a mixture of gases is equal to the sum of the specific polarizations of the components, and consequently the dielectric permittivity of the mixture is equal to [6]

$$\varepsilon = 1 + 4\pi \sum_i n_i \alpha_i\,, \tag{15}$$

where $\alpha_i = \alpha_i^{el} + P_{0i}/3kT$. Thus, for small impurities

$$\varepsilon_1(\bar{r}, t) = 4\pi \sum_i n_i^{(im)}(\bar{r}, t)\,\alpha_i\,. \tag{16}$$

In order to determine ε_1, we should know the profile of the impurity concentration upon loss of sealing of the liquid-metal loop. For this, we will solve the corresponding diffusion problem:

$$\begin{gathered}\frac{\partial n}{\partial t} = D\Delta n\,, \quad n(x, y, z, 0) = 0\,, \quad \frac{\partial n}{\partial x}(d, y, z, t) = 0\,, \\ n(x, y, -l/2, t) = n(x, y, l/2, t)\,, \\ -D\frac{\partial n}{\partial x}(0, y, z, t) = J\delta(y - y_0)\,\delta(z - z_0)\,.\end{gathered} \tag{17}$$

Just as in the problem of electrostatics, we neglect boundary effects along the coordinate y, $(0, y_0, z_0)$ is the point at which the leakage took place. Below, for evaluation it will be assumed that $y_0 = z_0 = 0$.

The solution of (17) is sought in the form

$$n(x, y, z, t) = \sum_{m=-\infty}^{\infty} \int_{-\infty}^{\infty} \exp\left(i\frac{2\pi}{l}mz + iky\right) n_m(x, k, t)\, dk\,, \tag{18}$$

and then the functions $n_m(x, k, t)$ satisfy the boundary-value problem

$$\begin{gathered}\frac{\partial n_m}{\partial t} = D\left[\frac{\partial^2 n_m}{\partial x^2} - k^2 n_m\right], \quad n_m(x, k, 0) = 0\,, \\ \frac{\partial n_m}{\partial x}(d, k, t) = 0\,, \quad -D\frac{\partial n_m}{\partial x}(0, k, t) = \frac{J}{2\pi l}\,.\end{gathered} \tag{19}$$

Solution of Eq. (19) using the Laplace transform yields

$$n_m(x, k, t) = \frac{1}{2\pi i}\int_{\sigma - i\infty}^{\sigma + i\infty} \exp(pt)\, dp \,\frac{J\,\mathrm{ch}\,\mu_m(x - d)}{2\pi\, lD\mu_m\,\mathrm{sh}\,\mu_m d}\,, \quad \mu_m^2 = k^2 + \frac{p}{D}\,. \tag{20}$$

The integrand is a ratio of two generalized polynomials. Having applied the theory of residues, we will have

$$n_m(x, k, t) = \frac{J}{2\pi\, lD}\left[\frac{\mathrm{ch}\,k(x - d)}{k\,\mathrm{sh}\,kd} + \sum_{j=1}^{\infty} \frac{\mathrm{ch}\,\mu_m^{(i)}(x - d)}{p_m^{(j)}\,[\mu_m\,\mathrm{sh}\,\mu_m d]}\exp(p_m^{(j)} t)\right], \tag{21}$$

where $p_m^{(j)}$ are the roots of the equation

$$\mu_m \operatorname{sh} \mu_m d = 0 , \quad j = 1, 2, \ldots , \tag{22}$$

and, accordingly, the concentration of the impurity is equal to

$$n(x, y, z, t) = \frac{J}{2\pi l D} \sum_{m=-\infty}^{\infty} \exp\left(i \frac{2\pi}{l} mz\right) \int_{-\infty}^{\infty} \exp(iky)\, k\, dk \left[\frac{\operatorname{ch} k(x-d)}{k \operatorname{sh} kd} + \right.$$

$$\left. + \sum_{j=1}^{\infty} \frac{\operatorname{ch} \mu_m^{(j)} (x-d)}{p_m^{(j)} [\mu_m \operatorname{sh} \mu_m d]} \exp(p_m^{(j)} t)\right] . \tag{23}$$

To determine the capacitance of the capacitor we substitute the obtained solution (23) into Eq. (14) and, performing the corresponding calculations, we obtain

$$\frac{C - C_0}{C_0} = \frac{\Delta C(\tau)}{C_0} = Af(\tau, \delta) , \tag{24}$$

$$f(\tau, \delta) = \int_{-\infty}^{\infty} \sin \frac{\eta\delta}{2} d\eta \left[\frac{1}{2\eta \operatorname{sh}^2 \eta}\left(1 + \frac{\operatorname{sh}^2 \eta}{2\eta}\right) - \frac{1}{\eta^3} \exp(-\eta^2 \tau) - \right.$$

$$\left. - 2 \sum_{n=1}^{\infty} \left(\frac{\eta \exp(-(\pi^2 n^2 + \eta^2)\tau)}{(\pi^2 n^2 + \eta^2)^2}\right)\right] , \tag{25}$$

where the following dimensionless quantities are introduced:

$$kd = \eta ; \quad \tau = \frac{D}{d^2} t ; \quad \frac{L}{d} = \delta ; \quad A = \frac{2\alpha J d}{\varepsilon_0 S D} .$$

A change in the capacitance entails a change in the charge of the capacitor and, consequently, a current appears in the chain:

$$J(t) = U \frac{dC}{dt} .$$

With account for Eqs. (24) and (25) we will have

$$\frac{J(t)\, d^2}{U C_0 D} = i(\tau) = A f_1(\tau; \delta) ,$$

$$f_1(\tau, \delta) = \int_{-\infty}^{\infty} \sin \frac{\eta\delta}{2} d\eta \left[\frac{1}{\eta} \exp(-\eta^2 \tau) + 2 \sum_{n=1}^{\infty} \frac{\eta \exp(-(\pi^2 n^2 + \eta^2)\tau)}{\pi^2 n^2 + \eta^2}\right] . \tag{26}$$

The parameter $\delta >> 1$, and for $f_1(\tau, \delta)$ the following approximate formula is valid:

$$f_1(\tau, \delta) = \frac{\pi}{2} \operatorname{erf}\left(\frac{\delta}{4\sqrt{\tau}}\right) . \tag{27}$$

The results obtained make it possible to evaluate the sensitivity of the method proposed. From general expressions (24)-(26), for the relative change in the capacitance under transient conditions we have

$$\frac{\Delta C}{C_0} \approx \frac{\alpha\delta}{\varepsilon_0 \sqrt{D}} \frac{J}{S} \sqrt{t} .$$

With allowance for the fact that $\alpha \approx 10^{-24}$ cm^3, $\varepsilon_0 \approx 1$, $D = 0.1-0.5$ cm^2/sec, $\delta \approx 10^3$,

$$\Delta C/C_0 \approx 10^{-21}(J/S)\sqrt{t}.$$

$$\frac{\Delta C}{C_0} \approx 10^{-21} \frac{J}{S} \sqrt{t} .$$

Since the change in the capacitance is limited by the maximum possible change in the dielectric constant of the gaseous medium ($\sim 10^{-4}$), consequently, for the sensitivity of the method we have the estimate

$$\frac{J}{S}\sqrt{t} \le 10^{17} \frac{\text{particles}}{\text{cm}^2 \cdot \text{sec}^{1/2}}, \tag{28}$$

where J/S is the mean flux of atoms of the working body in the interelectrode space of the capacitor.

NOTATION

q, charge; U, voltage; φ, electric-field potential; σ, density of the surface charge; C, capacitance; τ, t, time; x, y, z, coordinates; R_1, R_2, radii of the plates of the cylindrical capacitor; ε, dielectric permittivity; α, polarizability; α^{el}, electronic polarizability; P_0, intrinsic dipole moment of the atoms; d, l, L, geometric dimensions of the system; S, area of a plate; D, diffusion coefficient; n, concentration; J, number of particles entering the system per unit time; I, i, current.

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